EAS-301

B. Tech.

(SEM. III) (ODD SEM.) THEORY EXAMINATION, 2014-15

MATHEMATICS - III

Time : **3** Hours]   [Total Marks : **100**

Note : Attempt all questions. All questions carry equal marks.

1   Attempt any two parts of the following :   2×10=20

(a)   State the necessary and sufficient conditions for a function f(z) of a complex variable z, to be analytic in a region. Prove that

$$\nabla^2 \left\{ \left| f(z) \right|^n \right\} = n^2 \left| f(z) \right|^{n-2} \left| f'(z) \right|^2$$

stating the restrictions on f(z) .

(b)   State Cauchy's theorem for an analytic function. Verify it by integrating the function $f(z) = z^3 + iz$ along the sides of the rectangle with vertices (0,0), (2,0), (2,2) and (0,2).

(c) Evaluate the following integrals :

(i) $\int_0^\infty \dfrac{\sin x}{x}\, dx$

(ii) $\int_0^{2\pi} \dfrac{\cos 3\theta}{(5-3\cos\theta)^4}\, d\theta$

2 Attempt any two parts : 2×10=20

(a) Define the coefficients of skew ness and kurtosis of a distribution.
Find the coefficients of the skew ness and kurtosis of the distribution given by

$$f(x)=\begin{cases} e^{-x} & \text{for } x\geq 0 \\ 0 & \text{other wise} \end{cases}$$

(b) Fit a least – square parabola having the form $y = a + bx + cx^2$ to the following data :

x :	1.2	1.8	3.1	4.9	5.7	7.1	8.6	9.8
y :	4.5	5.9	7.0	7.8	7.2	6.8	4.5	2.7

(c) Define the coefficients of regression and correlation. Compute the correlation coefficient between the heights x of fathers and the heights y of their sons where x and y are given in the following table :

x (in cm)	160	165	170	175	180	185	190
y (in cm)	157	163	165	168	175	181	183

3 Attempt any two parts : 2×10=20

(a) Find the moment generating function of a random variable X that is binomially distributed.

(b) Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective, by using (a) the binomial distribution (b) the Poisson approximation to binomial distribution.

(c) In a blade manufacturing factory 1000 blades are examined daily. Draw the np- chart for the following table and examine whether the process is under control?

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defective blades	9	10	12	8	7	15	10	12	10	8	7	13	14	15	16

4 Attempt any two parts : 2×10=20

(a) Derive the Newton-Raphson's formula for finding the root of a function. Using this formula find a real root of the equation

$x^3 + 2x^2 + 10x - 20 = 0$

up to TEN iterations

(b) Find the Newton's forward interpolation polynomial for the following data :

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	5	12	17	21	27	29	35	40	50	58

Find f(5.7).

(c) Find a Lagrange's interpolation polynomial for the following data :

x	2	5	10	14	17	25
$f(x)$	5	11	21	27	35	47

Find the value of f (8).

5   Attempt any two parts :   2×10=20

(a) Solve the following system of linear equations by Gauss-Seidel method

$10x + y - z = 13$
$x + 10y + z = 29$
$2x + y + 10z = 41$

Correct up to two decimal- places

(b) Derive the formula for $\frac{3}{8}$ – Simpson's rule for the numerical integration. Using this rule,

evaluate $\int_0^6 \frac{e^x}{1+x} dx$

(c) Using Runge- Kutta method of order 4, find y for $x = 0.1, 0.2, 0.3$ given that $\frac{dy}{dx} = (x+y)y$ and $y = 1$ at $x = 0$.